invested in that fund, whatever is optimal for the fund is optimal for you," he points out, adding that "perhaps the crucial step is to start using the concept of objective optimality." This level of risk truly maximizes the return on an investment, not the level that seems like a good idea to a gung-ho fund manager angling for a massive reward.

Meanwhile, Foley suggests that the world financial system needs stronger controls on exchange rates. Something upon those lines was suggested by John Maynard Keynes at the 1944 Bretton Woods Conference, in which the allied nations made arrangements for global finances after World War II. But the United States was unwilling to give up the sovereignty that such controls would require—leading to the weaker World Bank and International Monetary Fund. This, says Foley, contributed to the problems of today, because it meant that the dollar became the de facto global currency and the United States found itself managing global demand, meaning that when its economy began to quake, the whole globe felt the aftershock.

Even with such a system, rebuilding isn't going to be easy. The markets may have moved into a new stable state: "It's a rather tough situation for policymakers to deal with," says Foley. "The good equilibrium may no longer be there at all." And even though many commentators and policymakers are calling for tighter regulation of financial markets, the appropriate stringency of regulation isn't at all clear. "If you regulate for the worst case, you over-regulate. But if you regulate for the normal case, you don't protect the system from collapse."

Regardless of what fixes we attempt, we need new ways to monitor and understand the system, says Farmer. The simple models of conventional economics are not up to the job. "We've got no model that deals with, for example, the fact that the financial system affects the production sector of the economy, yet that's what's causing the recession," he says. And taking complexity out of the markets isn't an option. Sophisticated financial instruments are here to stay, and can be a force for good. "We're not going to go back to banking in gold," he says. The only thing we can do, he concludes, "is to recognize the complexity and tackle it head on." ◀

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On Time and Risk

BY OLE PETERS

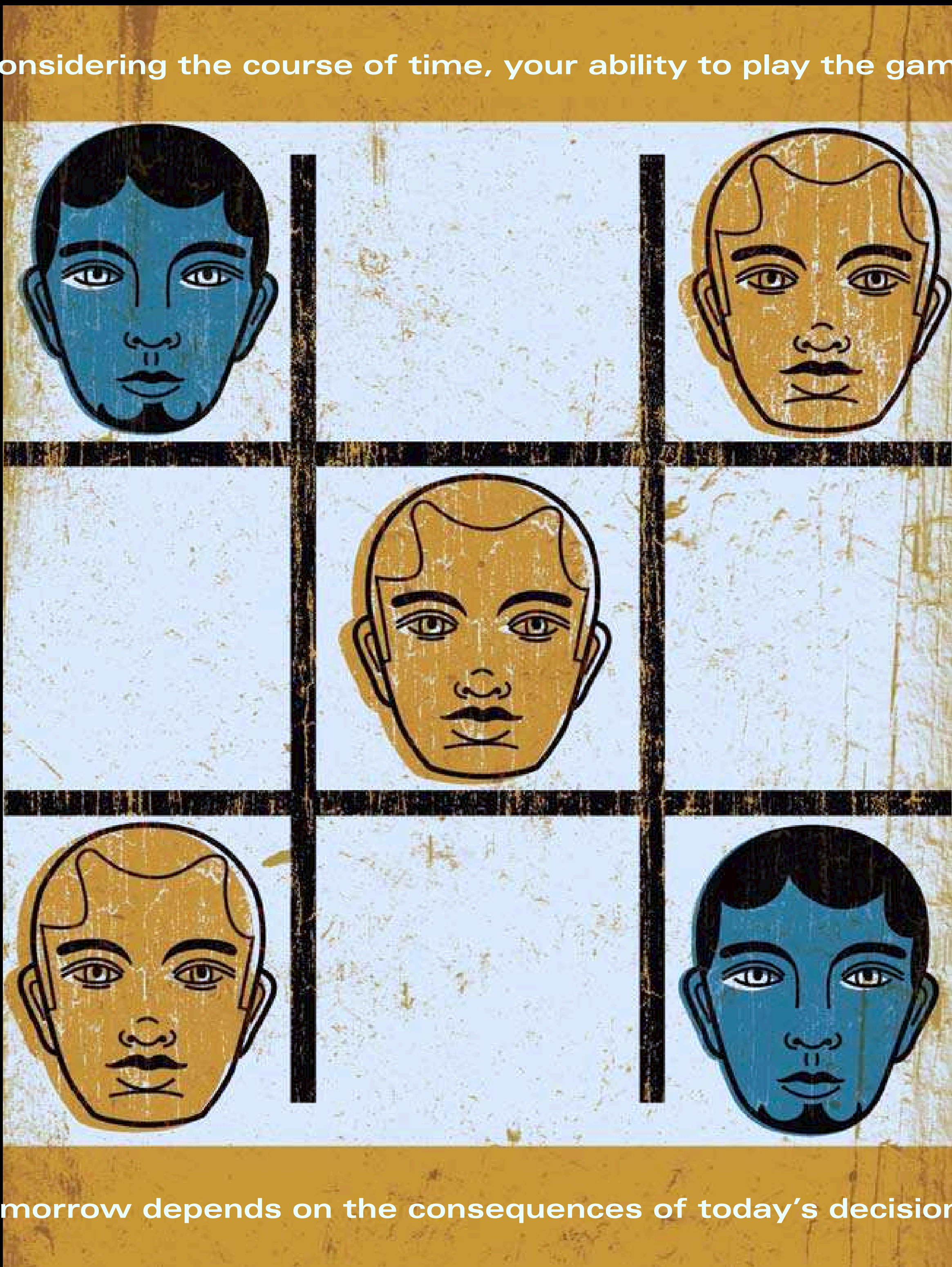
Let's say I offer you the following gamble: You roll a dice, and if you throw a six, I will give you one hundred times your total wealth. Anything else, and you have to give me all that you own, including your retirement savings and your favorite pair of socks. I should point out that I am fantastically rich, and you needn't worry about my ability to pay up, even in these challenging times. Should you do it?

The rational answer seems to be "yes"—the expected return on your investment is 1,583 1/3% in the time it takes to throw a dice. But what's your gut feeling? Perhaps you are quite happy with your present situation; maybe you own a house and a nice car and a private jet—would you be one hundred times happier if you were one hundred times richer? And how much less happy would you be if you suddenly had nothing?

This example illustrates a common flaw in thinking about risky situations, one that can make us blind to excessive risks and which appears to have been a factor in the financial markets in recent years. As we will see, the calculation of the enormous expected return essentially assumes that you have dealings with parallel universes. Consequently, financial models can fall prey to the assumption that traders will regularly visit the parallel universe where everything comes up sixes. An analysis of risk and return that prohibits such eccentricities gives rather different answers. We will start with an outline of the classical treatment of risky problems, then offer an alternative, and finally discuss the practical consequences of both perspectives.

Daniel Bernoulli, the man who explained why helicopters fly a few hundred years after Leonardo da Vinci drew them and a few hundred years before they took to

Considering the course of time, your ability to play the game



tomorrow depends on the consequences of today's decisions.

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Diogenes by John William Waterhouse. Oil on canvas (1882). Living in a tub on the streets of Athens in the 4th–3rd century B.C., Diogenes was a beggar who made a virtue of extreme poverty.

the skies, contemplated pretty much our gamble, when, in 1738, he offered his answer to what economists now call the St. Petersburg paradox. The paradox asks how much a rational person should pay for a lottery ticket that offers a very low chance of a tremendous win.

He pointed out that mathematics alone does not capture the situation. It produces numbers for us like 1,583 1/3%, but it cannot give those numbers meaning, for the fundamental reason that how much I own is irrelevant—what matters is what use my possessions are to me. I might require an expensive, life-saving operation next week, which limits my ability to take risky gambles. Or my name could be Diogenes, and when offered riches I yawn and mumble something about shade and sun, wave a hand and turn around in my tubular abode. St. Exupéry's Little Prince comes to mind, who stares in bewilderment at the business man who is counting the stars that he owns.

Bernoulli argued intuitively that the increase in the usefulness—utility—of my total wealth from a small gain should be inversely proportional to the wealth I already have. If I'm rich already, another dollar won't make much difference (although he also acknowledges exceptions, such as a rich man in prison whose utility increases more due to the extra ducats required to buy his freedom than that of a poorer man given the same amount). Mathematically expressed, this assumption amounts to a so-called logarithmic utility function. Utility functions had already been established before 1738 as a concept to reflect risk preferences and became the standard answer to problems where investments are characterized by an expected return and an uncertainty in that return.

Bernoulli's answer, logarithmic utility, reconciles the mathematics with our gut feeling—the expected utility (or logarithm) of your wealth after playing my game is negatively infinite, a strong warning against taking the gamble. But because his perspective is intuitive, it is vulnerable to modifications. Arguing on the basis of usefulness, different types of utility functions, designed to include rare exceptions like the rich prisoner, are no less valid than the logarithm he proposed. After all, these functions are supposed to reflect personal choices and circumstances. Thus, invoking the individuality of human beings, Bernoulli's peers emphasized that the full treatment of the problem is outside the realm of reason. But this sounds more like a cheap excuse than an answer to the problem—and what's more, an excuse to choose a utility function that gives the answer I want.

A less vulnerable perspective that, strangely, remained on the fringes of economic theory, was

If you find yourself in this situation, by all means, play the game. But if you're a mere mortal, I'd advise you not to do it.

pointed out 218 years after Bernoulli's treatment of the problem by John Larry Kelly in 1956. I offer you the same bet as before. This time, following Kelly, we will make do without utility and instead focus on the irreversibility of time. Since we're considering a situation with randomness, we're interested in some expected, or average performance. Playing the game repeatedly, we might expect the performance over many rounds to converge to this average.

Why might we expect this? If I ask you to roll your dice 100 times and tell me how many sixes you got, your answer will be somewhere around 17. Alternatively, we could measure the expected number of sixes by giving one dice to each of 100 people and let everyone roll once. In this instance, we will find a similar number of sixes—again, around 17. Whether we look at a time average (you rolling your dice many times) or an ensemble-average (many people each rolling a dice once)—as the number of trials increases the fractions of sixes will converge to 1/6.

It seems trivial that the two differently computed averages should be the same—trivial enough for mathematical physicists to question it. Ludwig Boltzmann, in about 1884, coined the term "ergodic" for situations with identical time averages and ensemble averages. Not every situation is like this, however; there exist "non-ergodic" situations as well, and these are often as counterintuitive as the ergodic situations seem trivial.

So do we have to be more careful when we talk about expected returns and average performances? There are two averages, not one—two ways of characterizing an investment, two quantities with different meanings. Let's consider each in turn, ask which one is relevant in our case, and see if they are identical.

First the ensemble average: When economists, or Bernoulli, speak of "expected return," they typically mean an average that is calculated as the sum over all possible outcomes, weighted by the probabilities of these outcomes. An example is the 1,583 1/3% per round expected return of our game.

Probing a little deeper, we discover that this calculation uses the conceptual device of an ensemble of infinitely many identically prepared systems, or copies of our universe. The ensemble average simultaneously considers all possible paths along which the universe might evolve into the future. The fraction of systems from the ensemble that follows some scenario is the probability of that scenario, and summing the possible outcomes and weighted with their respective probabilities amounts to taking an average over all possible universes.


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Daniel Bernoulli, an 18th-century Dutch-Swiss mathematician, pioneered work in probability and statistics.

Herein lies the danger: If we don't actually play many identical games at once, then such an average only has practical relevance if it is identical to the quantity we're interested in, often the time average. There may be many possible paths from here into the future, but only one will be realized. In our game, you are risking your entire wealth, which obviously cannot be done many times simultaneously, so the ensemble average is not really the relevant quantity. Technically, it stems from a gedanken experiment involving other universes.

Now the time average: Perhaps it is identical to the ensemble average, and it doesn't matter which one we use. In other words we ask, is the situation ergodic? Considering the course of time, your ability to play the game tomorrow depends on the consequences of today's decisions, and next month's ability depends on the 30 daily outcomes in between. The ability of one player in the ensemble to play the game, on the other hand, does not depend on other players' luck. For this reason the ensemble average return is different from the time average—maliciously so: The time average performance of a single investment is always worse than the ensemble average. So unfortunately, the situation is not ergodic.

In our initial treatment of the game, the fact that I asked you to risk everything you own didn't impress the mathematics—it produced an expected return that seemed to strongly recommend playing the game. The reason this ensemble average didn't respond to the fact that you were most likely about to lose everything is this: The ensemble includes those few lucky copies of yourself whose enormous gains would easily make up for your likely loss.

Following Bernoulli, we reconciled the tempting expected return with our intuition by introducing utility. But this is not necessary—we simply need to recognize that we used an inappropriate average, implicitly treating the game as if we could interact with those parts of the ensemble that did not materialize (i.e., parallel universes) and realize the average return over all universes. If you find yourself in this situation, by all means, play the game. But if you're a mere mortal, I'd advise you not to do it. The time-average growth rate for this game, just like the expected logarithmic utility, is negatively infinite—if you don't believe me, play it a few times in a row. Instead of different changes in utility, the time perspective emphasizes that, as time goes by, we cut off different numbers of branches of potential universes reaching from the present into the future. The difference in perspective is subtle but has far-reaching consequences.

We've considered an extremely risky game for illustration, but none of the above arguments are specific to it. In general, the time perspective reveals an upper limit on risks that may be considered sensible. For example, suppose I offered you a similar but different game: You get to roll a dice and whatever you wager, I will give you 100 times your wager if you throw a six. This situation is different because you can hold back some of your wealth in case you lose. In fact, the time perspective will tell you to invest about 16% of your net worth and keep playing the game, adjusting the wager to that same fraction after every round. It also tells you that over time you will realize a growth rate of about 33% per round. Crucially, *if you choose to risk more than this, you will gain less* (of course you will also gain

Today's risk management often solely relies on investors specifying their risk preferences, or, synonymously, their utility functions, without explicitly considering the effects of time.

less if you risk less than 16% of your wealth).

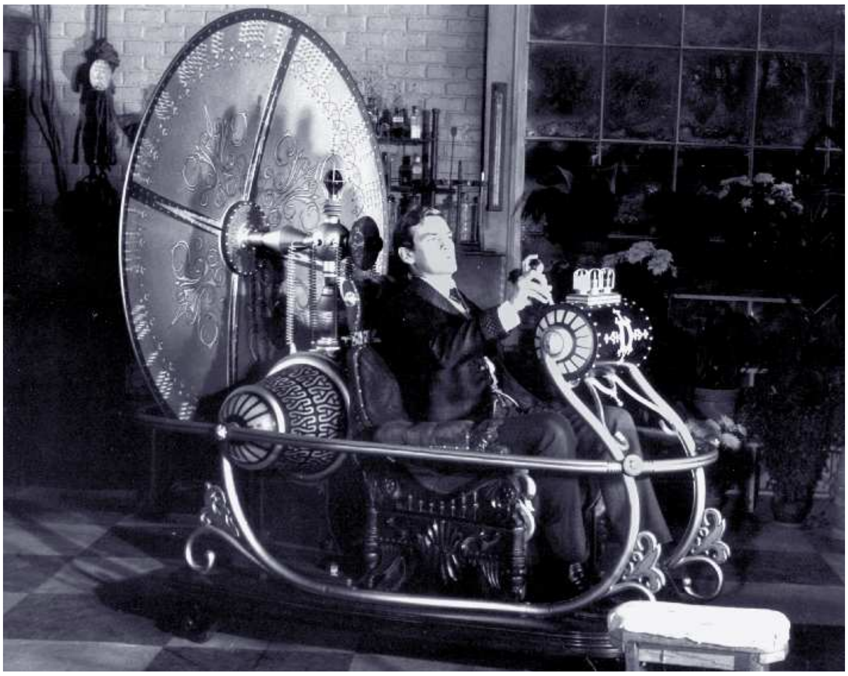
A time-based approach provides insights into how to regulate credit rationally: how much an investment should be leveraged, the loan-to-value ratio at which a mortgage becomes a gamble, and the appropriate requirements for margins and minimum capital.

The literature on portfolio theory and risk management largely uses a combination of ensemble averages and utility, neglecting time or at best encapsulating its effects in a utility function. In this approach, time irreversibility, the unshakable physical motivation for refraining from excessive risk, is replaced by arbitrarily specifiable risk preferences. Following the establishment of the corresponding academic framework (roughly from the 1970s), regulatory constraints that were largely based on common sense were progressively loosened.

In an investment context, the difference between ensemble averages and time averages is often small. It becomes important, however, when risks increase, when correlation hinders diversification, when leverage pumps up fluctuations, when money is made cheap, when capital requirements are relaxed. If reward structures—such as bonuses that reward gains but don't punish losses, and also certain commission schemes—provide incentives for excessive risk, problems arise. This is especially true if the only limits to risk-taking derive from utility functions that express risk preference, instead of the objective argument of time irreversibility. In other words, using the ensemble average without sufficiently restrictive utility functions will lead to excessive risk-taking and eventual collapse. Sound familiar?

Considerations of time alone cannot capture an investor's or a society's risk preferences. These preferences will always depend on individual circumstances and include motivations, for example moral motivations, that are indeed beyond the reach of mathematics. But time considerations do place objective upper bounds on advisable risks, and go a long way towards rationalizing our intuitions.

Today's risk management often solely relies on investors specifying their risk preferences, or, synonymously, their utility functions, without explicitly considering the effects of time. My bank asked me the other day what risk type I am, apparently expecting a reply like "I like a good gamble," or "I always wear my bicycle helmet." When I replied with a statement regarding time and answered, truthfully, that I'm the type who likes to see his money grow fast, they thought I was joking. ◀



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From the 1960 movie, *The Time Machine.* Most current risk strategy acts as though there are many possible paths from the present into the future, but really only one will be realized.

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